LamiOri

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Text, mathematics, editing: Alessandro Beber, Apr. 2019

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1. Overview

Alessandra Lamio designed one of these origami structures (the `5:2` one) back in November 2013, while exploring corrugations based on square grids. This first design was not developed from a crease-pattern, but rather by working directly on pleated paper. Then, in April 2014, she discovered another such pattern by chance (the `3:1` one, a sunk variation of which was previously designed by Robert J. Lang[1]), while trying to fold the previous one by heart, as she did not have the original model at hand nor any notes about it. At that point it was clear that a whole family of origami corrugations was possible, by varying just two parameters, and she began a journey on trying to understand its rules, in collaboration with Alessandro Beber. It turned out that all the members of this family can fold flat, but they display different characteristics in terms of aspect, ratio, curvature, flexibility, and other mechanical properties.

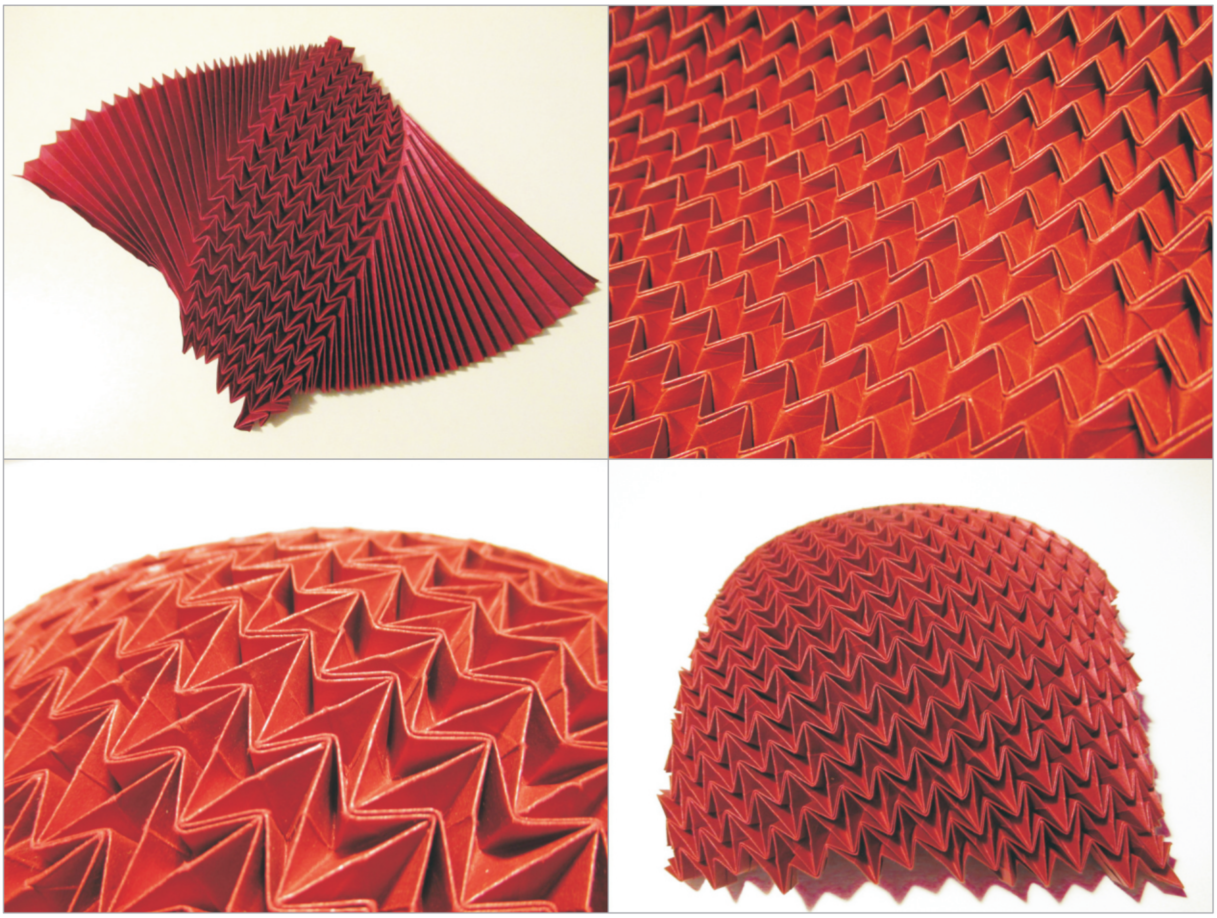


Figure 1: Various views of the original "LamiOri `5:2`", © Alessandra Lamio 2013

2. Structure

The structure of the origami corrugations in this family can be considered as derived from the classic “Waterbomb corrugation” (see [2] and [3]), which could be also called “LamiOri `2:1`” as we shall soon see. The basic *tile* is always a rectangle of height `h=2` units, and length `n 2` units, with valley folded edges. Such rectangles are filled in with four mountain folded angle bisectors (45° lines), an horizontal mountain fold line connecting their *intersections* (this line disappears in the limit case `n=h=2`, i.e. forming a *waterbomb base*), and two opposite small vertical valley folds in order to flat fold such intersections. Considering a single tile of length n, its *left border* will be placed at position `(0)`, its *right border* at position `(n)`, and its *inner intersections* at positions `(+1)` and `(n-1)`.

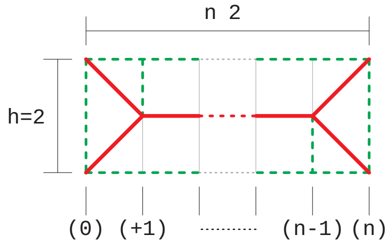


Figure 2: A single tile of length `n`.

Consecutive copies of these tiles are then arranged in rows, placed on a gridded paper, each row being offset of `s` units with respect to the previous one. We will call this amount `s` the *shift* or *step* of the pattern, and the resulting origami corrugation will be named “LamiOri `n:s`”.

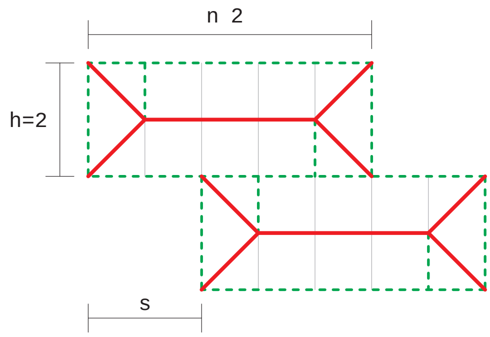


Figure 3: Two tiles of length `n`, offset `s` units.

Given tiles of length `n`, only certain shift amounts `s` are possible. Let’s see how this works!
The left and right borders of a given tile must extend upwards and downwards through the pleated paper, until they meet an inner intersection on each side in order to stop and flatten. In particular, the *extended border* must meet a `(+1)` intersection upward and an `(n-1)` intersection downward or vice-versa, due to the symmetry of the tile itself. It turns out that this condition is satisfied if and only if:

$$i \quad \mathbb{N} : (i \cdot s \quad +1 \ (\mathrm{mod}\ n)) \quad (i \cdot s \quad n-1 \ (\mathrm{mod}\ n))$$

or, more simply, if and only if `n` and `s` have no common divisors except `1`:

$$\neg \quad i \quad \mathbb{N},\ i \ 1 : i|s \quad i|n$$

If there was a common divisor, then an extended border would stop against another border, without ever reaching an intersection and preventing the whole structure to fold flat.

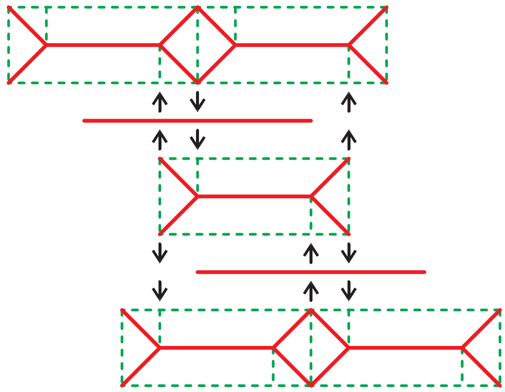


Figure 4: Extended borders must meet `(+1)` and `(n-1)` intersections while going upward and downward, in order to flatten the whole structure.

Furthermore, given the symmetry of the structure, if s_1 satisfies the previous conditions for a given `n`, then also s_2=`n-`s_1 will do, resulting in "LamiOri `n:`s_1" and "LamiOri `n:`s_2" being mirror versions of each other.
Thus, for `n=2...15`, the possible steps `s` are the following:

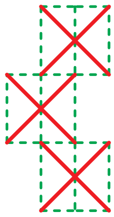
n	s	
2	1 [waterbomb]	
3	1	2
4	1	3
5	1, 2	3, 4
6	1	5
7	1, 2, 3	4, 5, 6
8	1, 3	5, 7
9	1, 2, 4	5, 7, 8
10	1, 3	7, 9
11	1, 2, 3, 4, 5	6, 7, 8, 9, 10
12	1, 5	7, 11
13	1, 2, 3, 4, 5, 6	7, 8, 9, 10, 11, 12
14	1, 3, 5	9, 11, 13
15	1, 2, 4, 7	8, 11, 13, 14

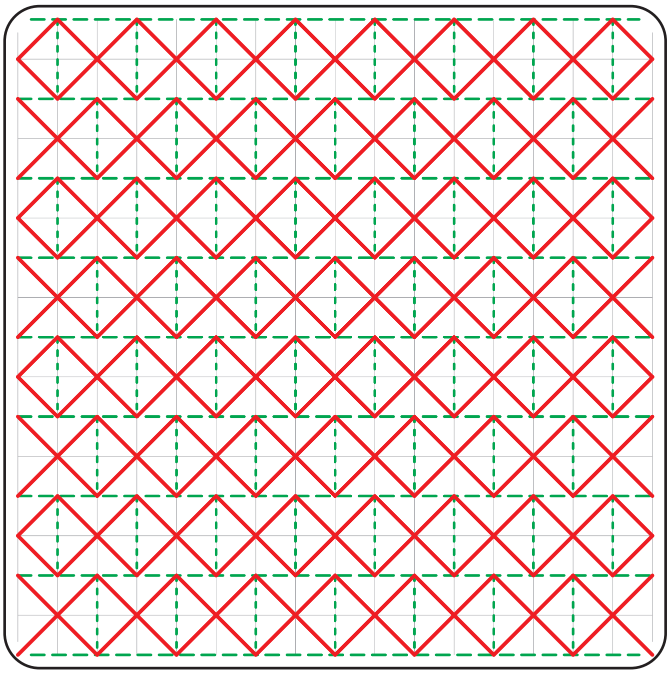
Table 1: possible offsets `s` for `n=2...15`.

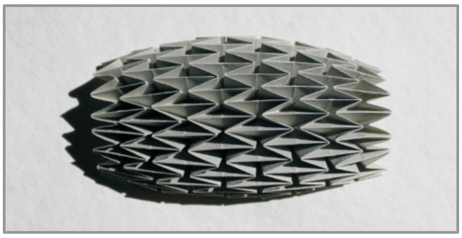
3. Examples

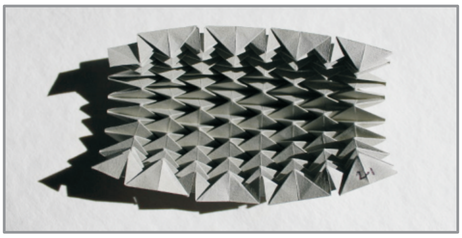
On the following pages there are examples of all the possible structures for `n=2...10`: they display different patterns onto their surfaces, and also behave differently in terms of ratio, strength, flexibility and other mechanical properties, the analysis of which could be the subject for a future research. Each example shows an arrangement of three tiles, a 16x16 grid crease pattern, and the folded result as seen from the front and back sides, also folded from a 16x16 grid.

LamiOri 2:1
[waterbomb corrugation]

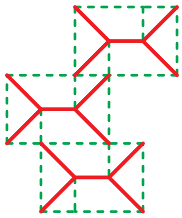


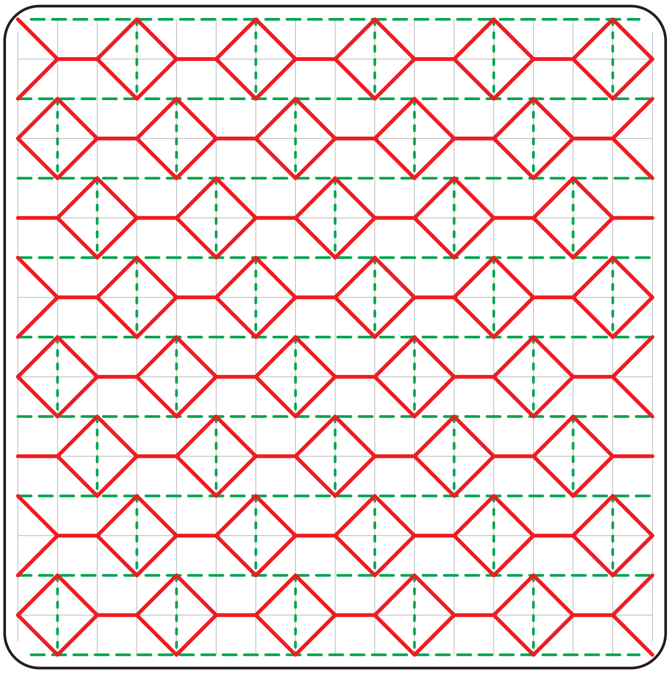


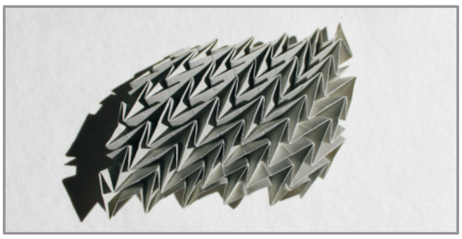


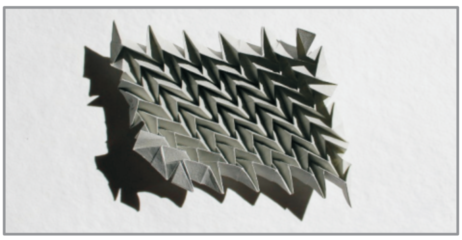


LamiOri 3:1

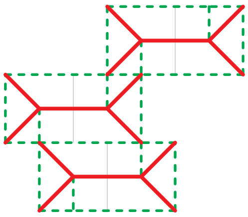


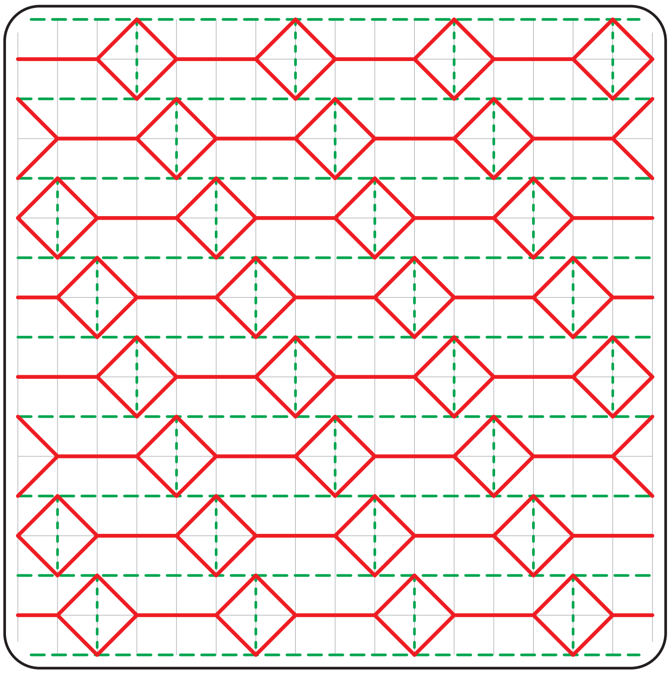


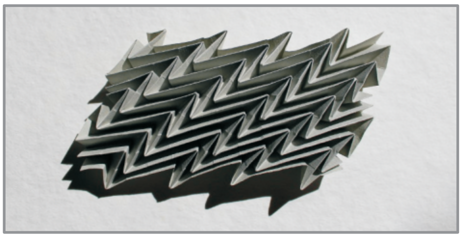


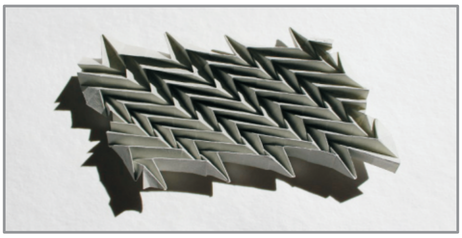


LamiOri 4:1

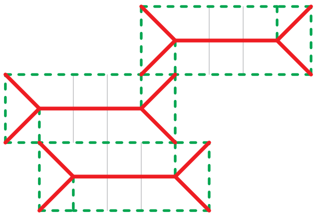


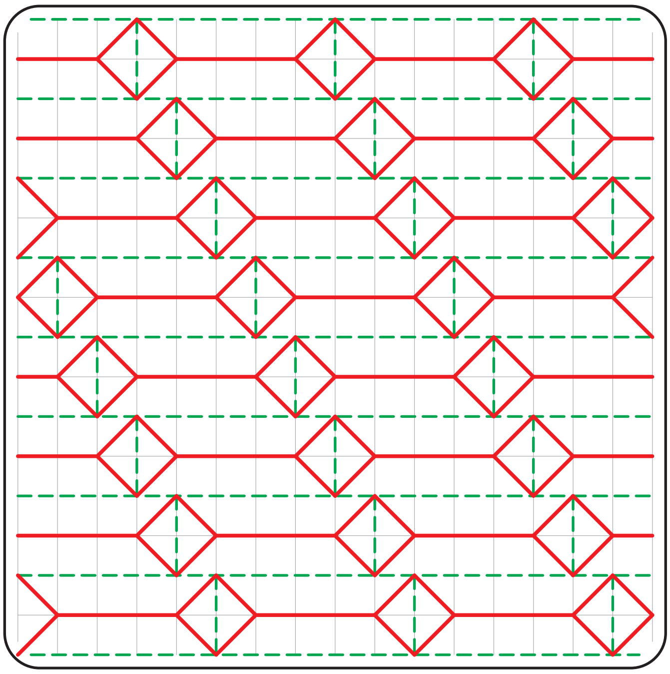


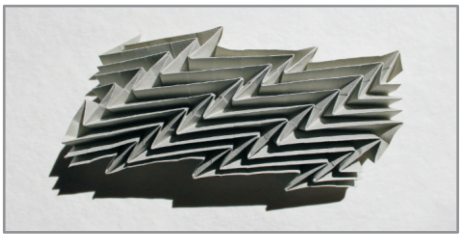




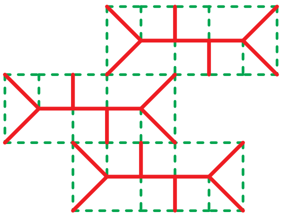
LamiOri 5:1

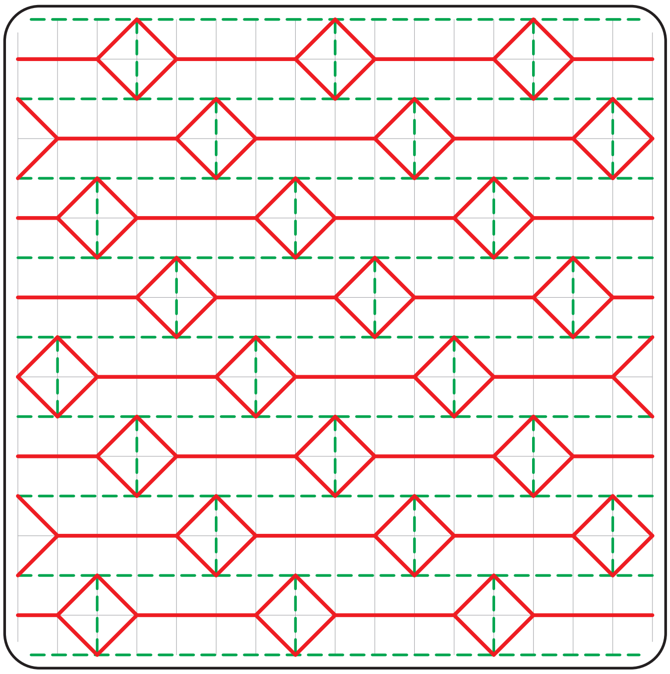


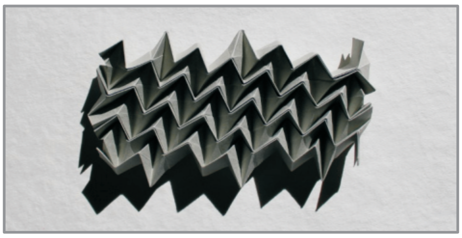




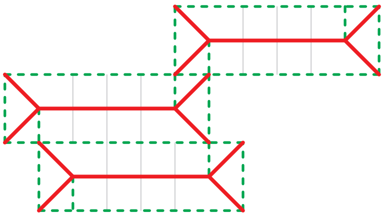
LamiOri 5:2

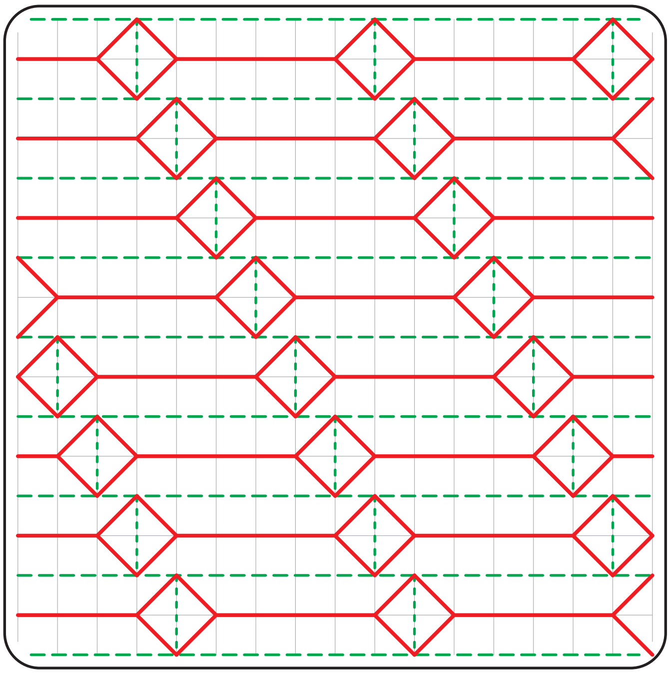


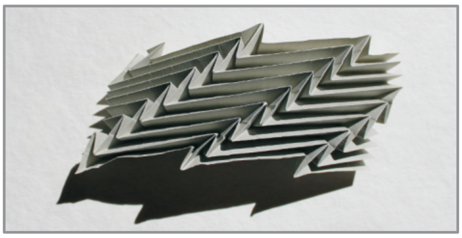


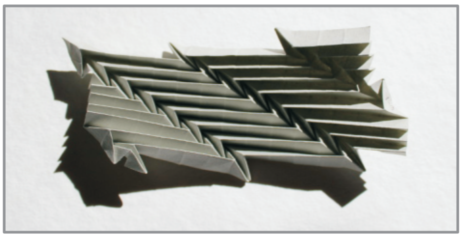


LamiOri 6:1

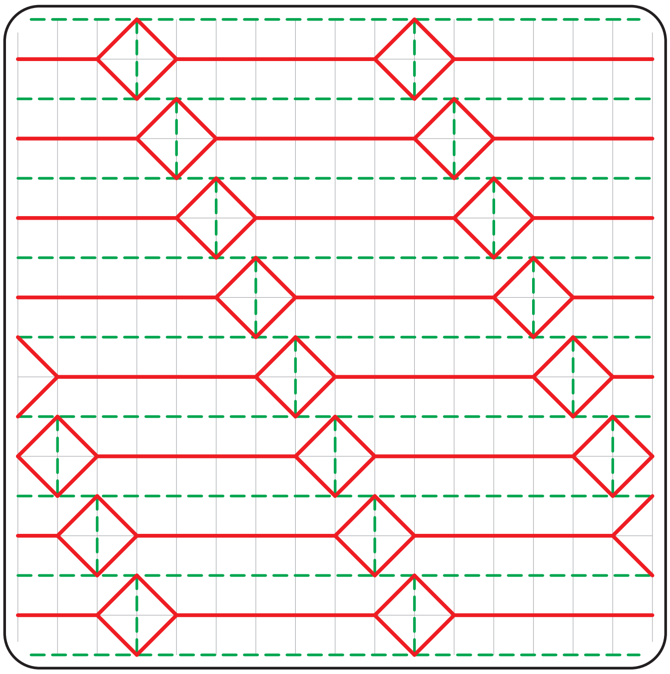


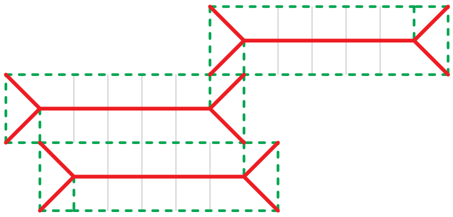


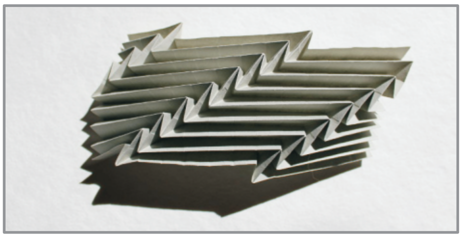


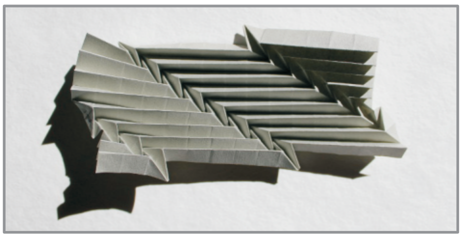


LamiOri 7:1

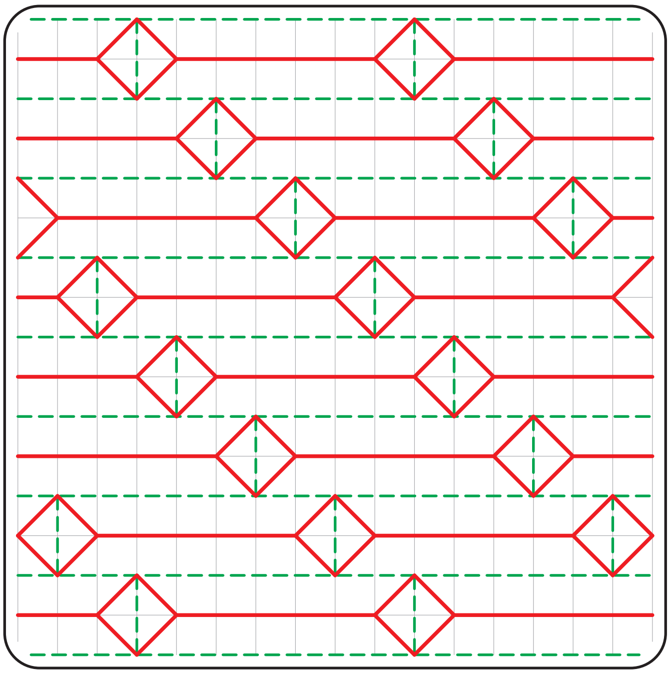


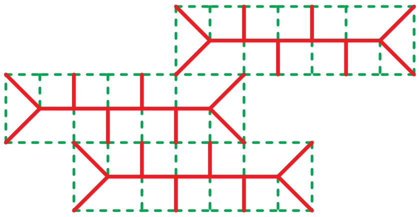


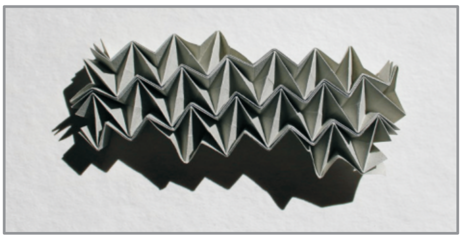


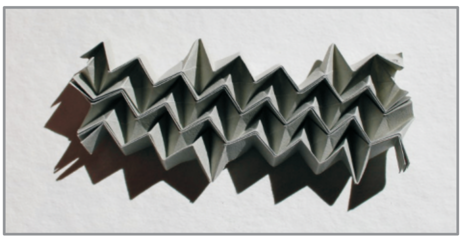


LamiOri 7:2

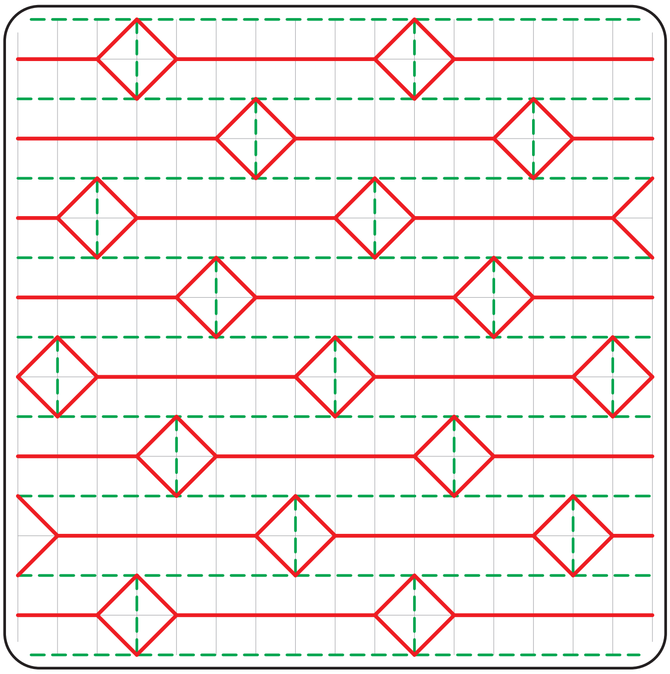


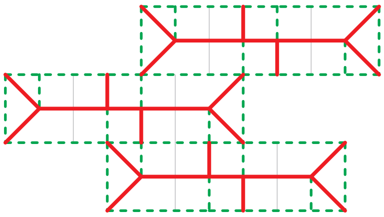


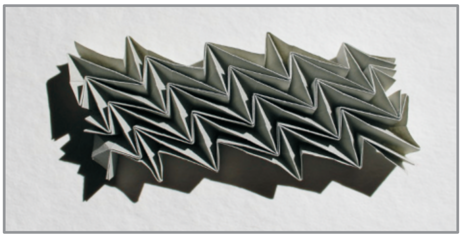


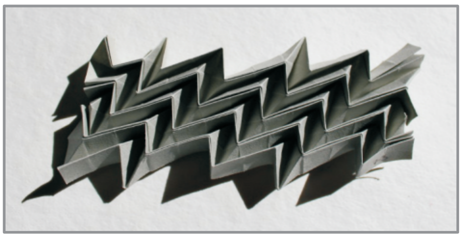


LamiOri 7:3

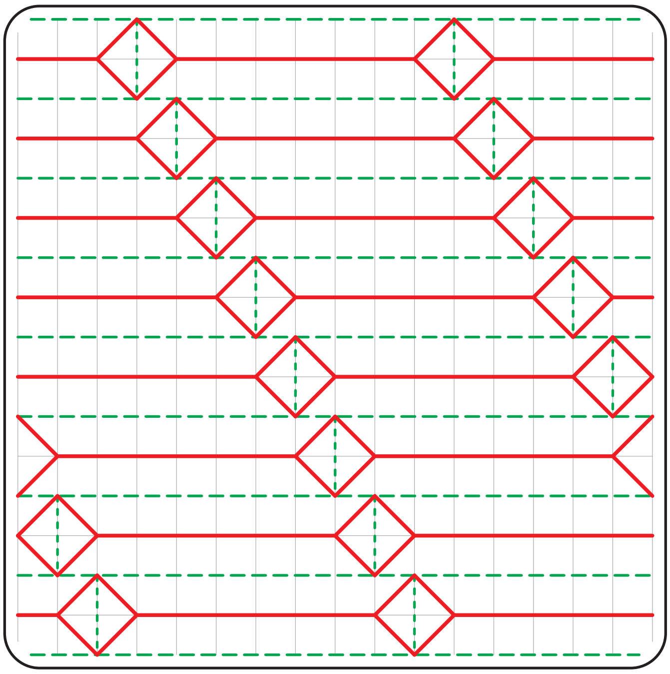


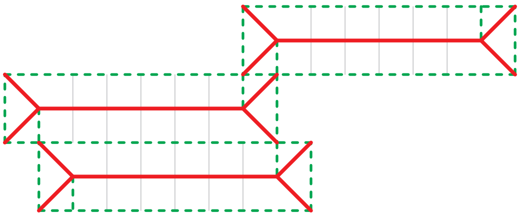


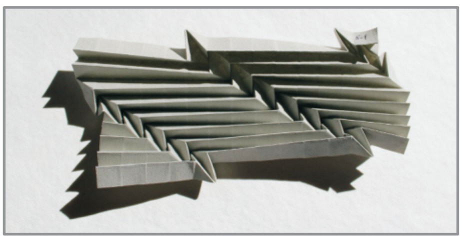




LamiOri 8:1







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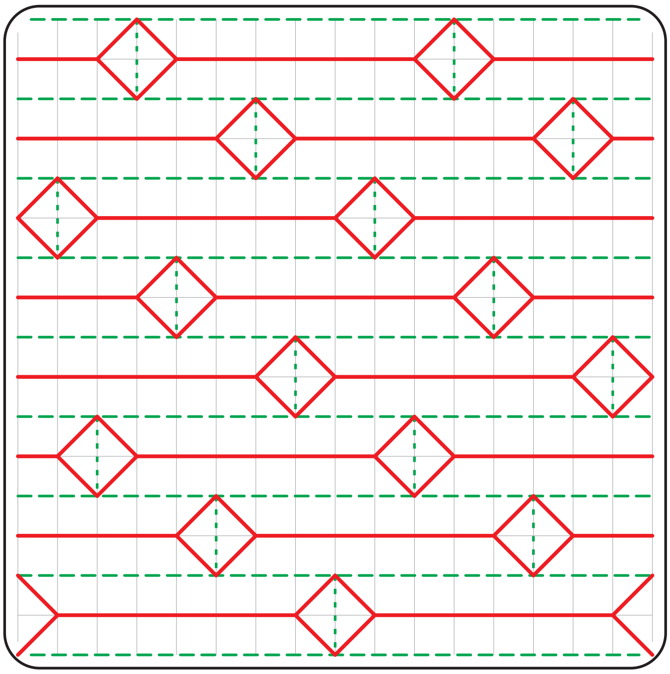
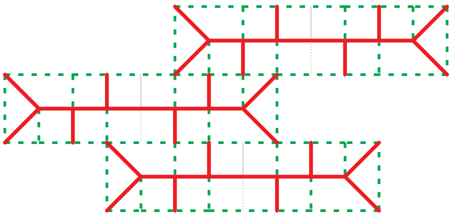
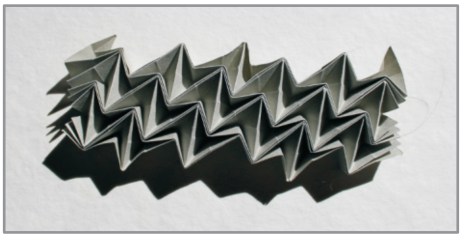
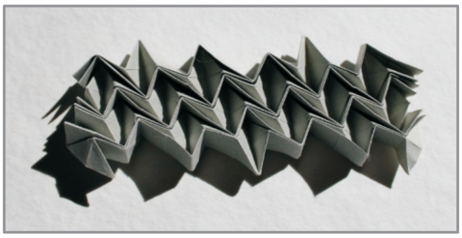
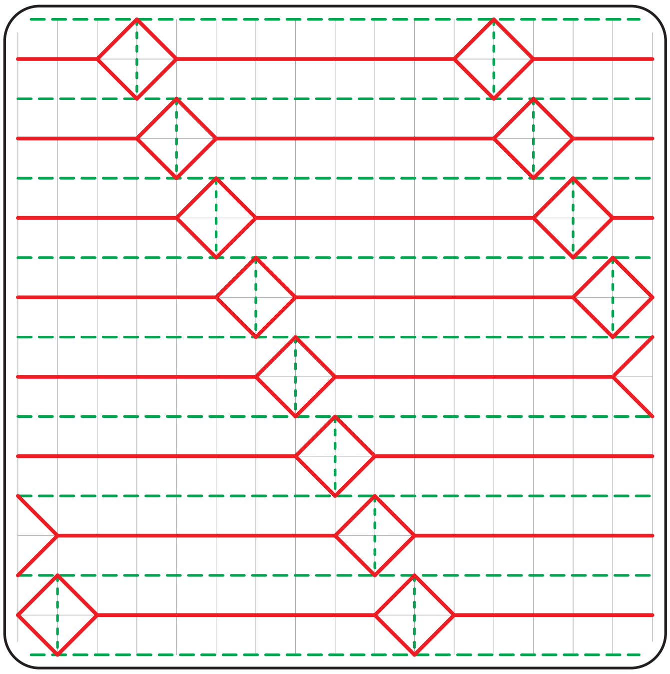


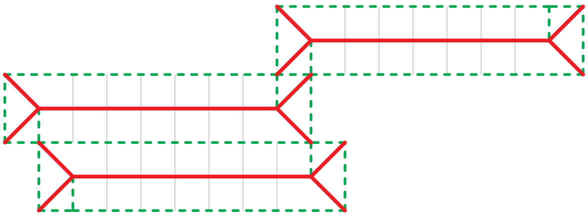


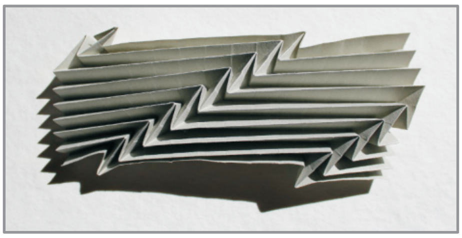


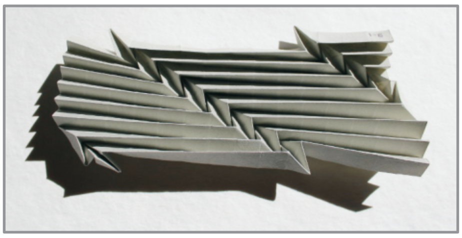


LamiOri 9:1

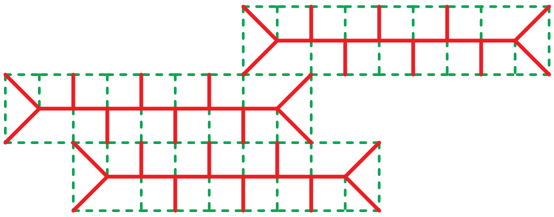


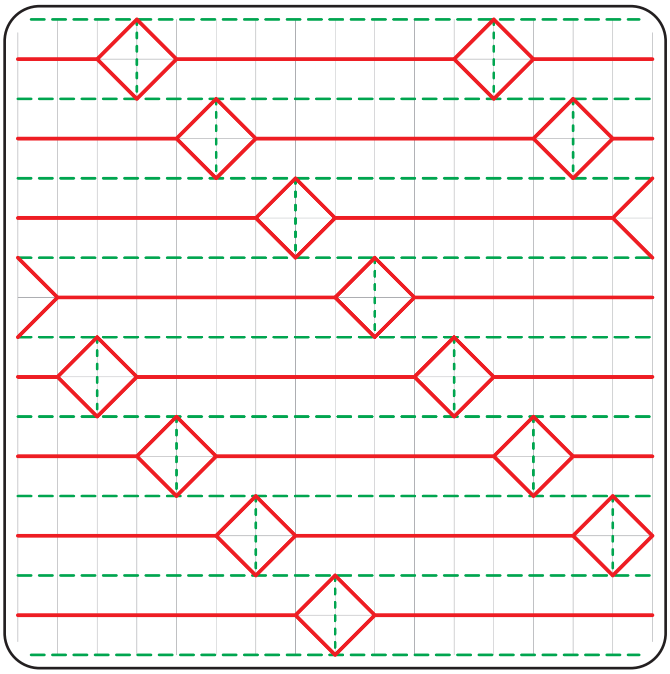


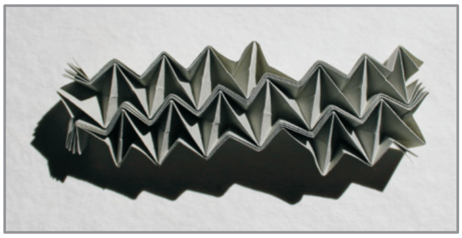


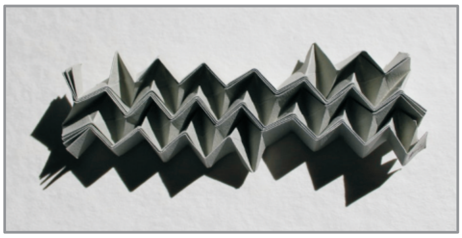


LamiOri 9:2









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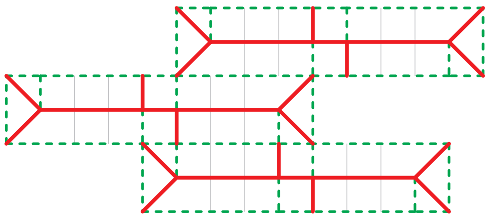
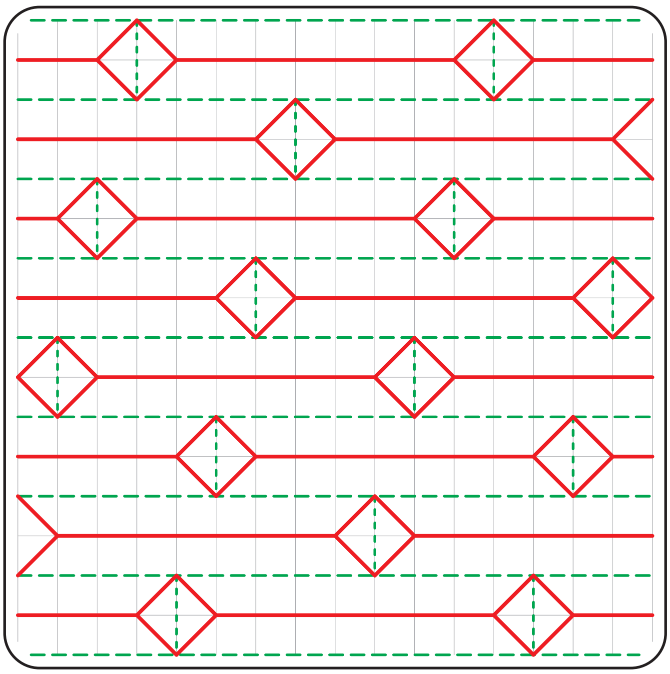
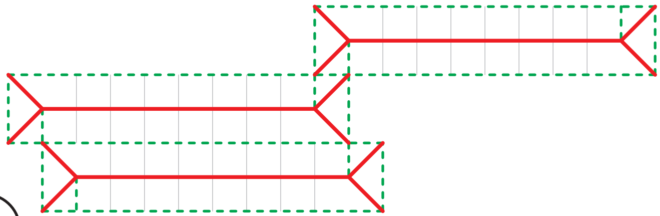
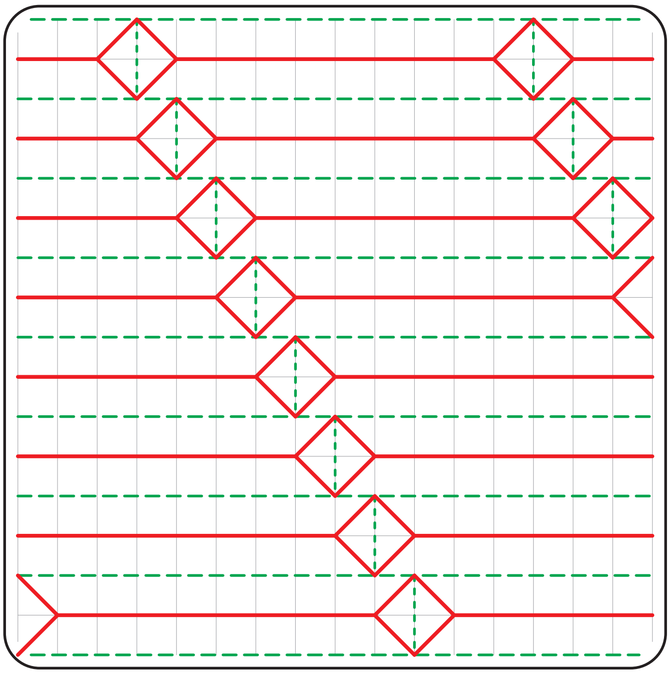
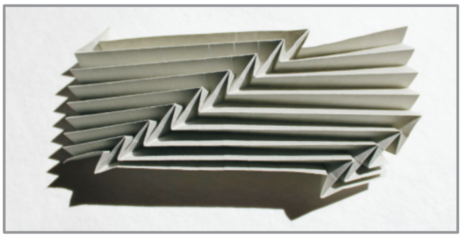


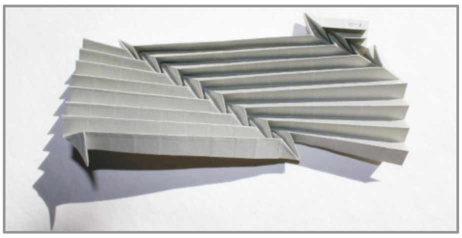


LamiOri 10:1

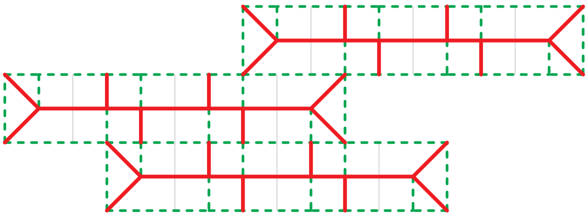


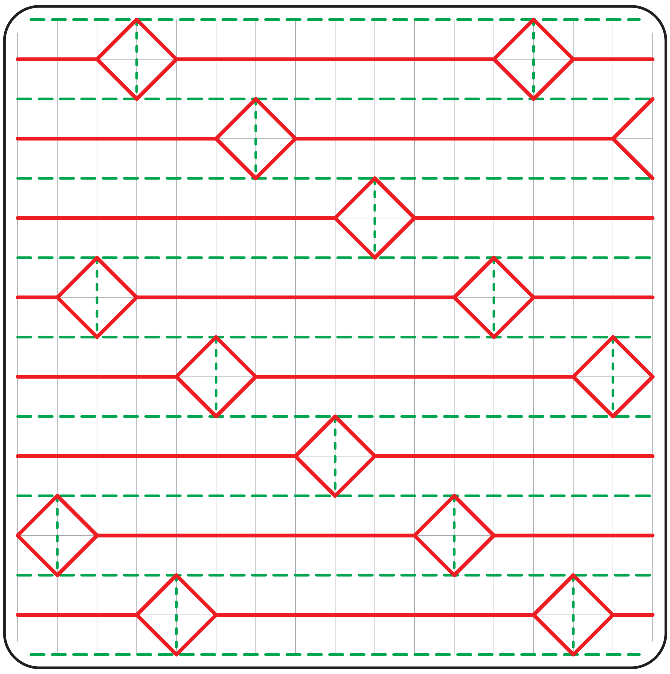


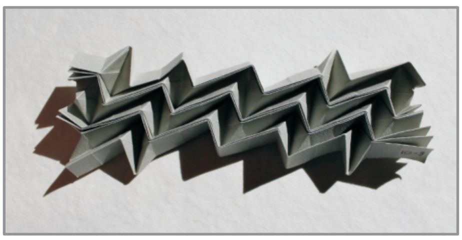




LamiOri 10:3







4. Patterns

By analyzing these origami structures through modular arithmetic it is possible to predict the aspect of their finished result, which can be grouped into three different sets (excluding the “Waterbomb corrugation” or “LamiOri 2:1”).

The first group displays Z-shaped molecules with straight horizontal edges, including all and only the patterns having n>2 and s=1.

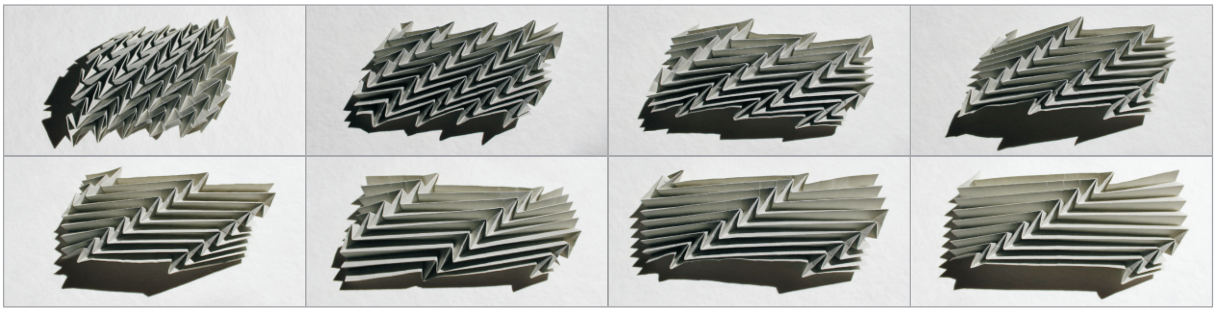


Figure 5: LamiOri patterns 3:1, 4:1, 5:1, 6:1, 7:1, 8:1, 9:1 and 10:1.

The second group displays almost equilateral triangle indentations (arrowhead-shaped concavities), including the patterns 5:2, 7:2, 8:3 and 9:2. This result is given by filling in all the positions between (0) and (n) with creases, by consecutively adding positive shifts s to (0) and negative shifts -s to (n), before stopping when reaching (+1) or (n-1).

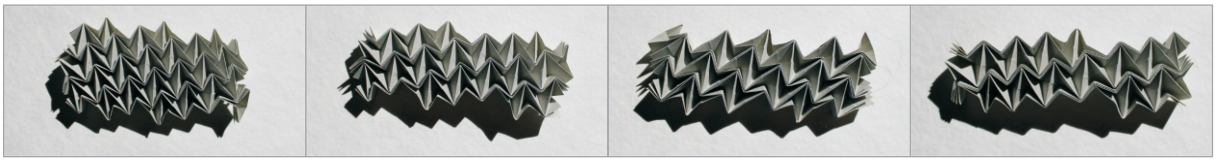


Figure 6: LamiOri patterns 5:2, 7:2, 8:3 and 9:2.

The third group displays sharper triangle indentations (longer arrowheads), including the patterns 7:3, 9:4 and 10:3. This can happen only when s 3, if position (+2) (and eventually some more following ones, as in “LamiOri 9:4”) are left free from creases. It means that consecutively adding positive shifts s to (0) and negative shifts -s to (n), some positions including (+2) and (n-2) are not reached before stopping at (+1) or (n-1).

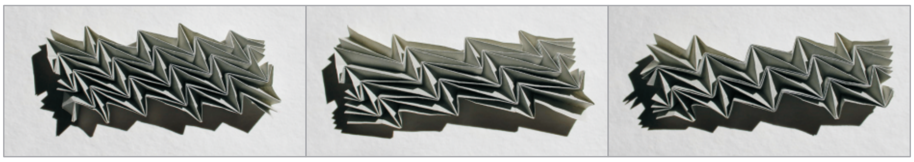


Figure 7: LamiOri patterns 7:3, 9:4 and 10:3.

You can also notice that the pattern 8:3 displays arrowhead-shaped indentations having opposite orientation with respect to the other members of the same group. This is because the extended border meets a (+1) intersection instead of an (n-1) one while going upward/downward. This may also happen with some member of the third group.

5. Further Possibilities

It is also possible to combine tiles of different lenghts in a sequence, as in the following artwork. In this case, the curvature and flexibility of the structure change along its surface; while `s=1` always works for such sequences, a general rule for determining a suitable shift `s` has yet to be worked out.

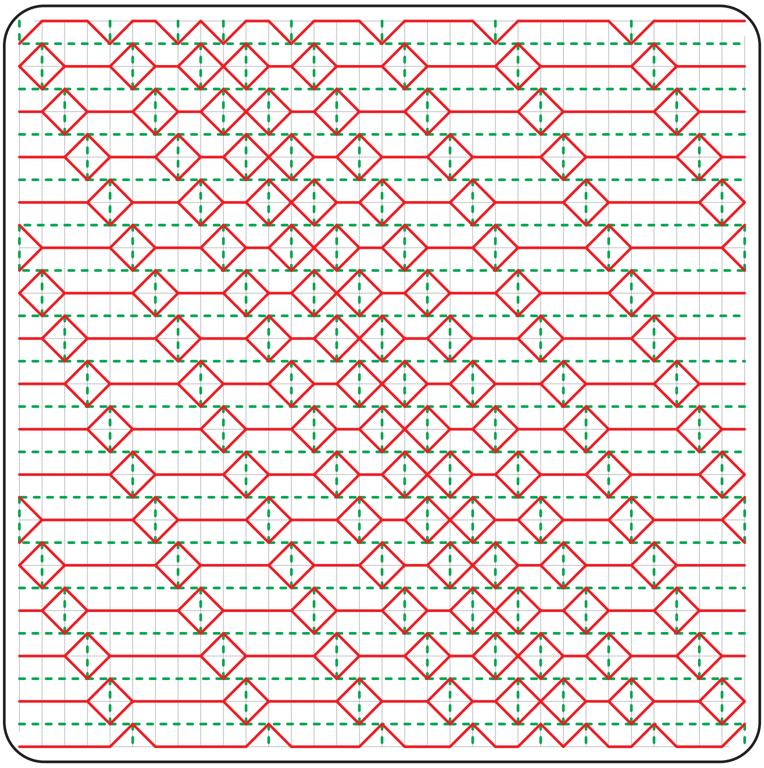


Figure 8: Crease pattern for an example of “Progressive LamiOri”.

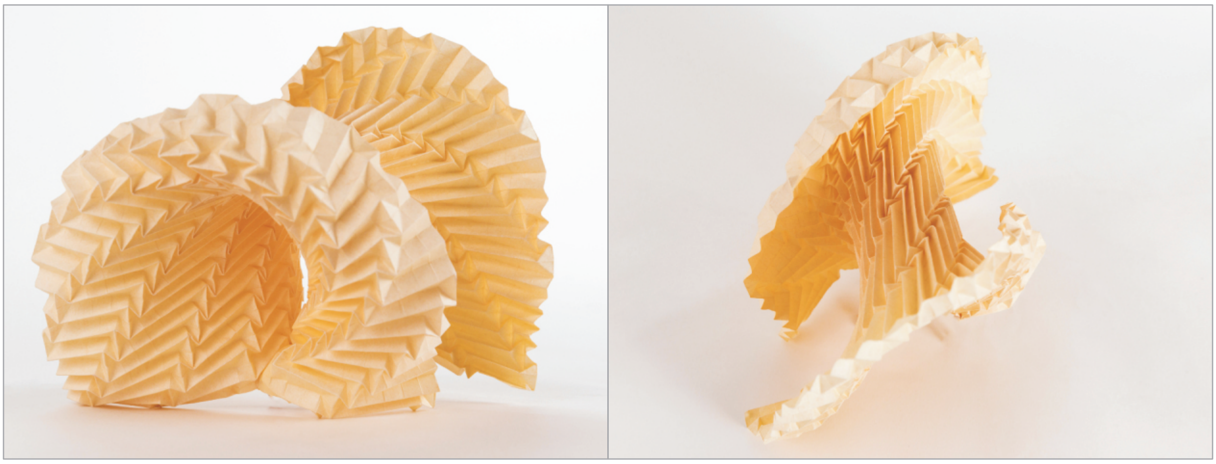


Figure 9: Folded result of another version of “Progressive LamiOri”.

Thanks to Matthew Gardiner for suggesting the title for this family of origami corrugations, during a discussion in Oxford, UK (September 2018) after the 7OSME conference.

References:
[1] Robert J. Lang, “Molecular Tessellation”, 2004, https://langorigami.com/artwork/molecular-tessellation/
[2] Shuzo Fujimoto, *Rittai Origami (3D Origami)*, p.68, Japan, 1976.
[3] “Waterbomb Tessellation”, in Robert J. Lang, *Twists, Tilings and Tessellations*, pp.164-174, CRC Press, 2018.